$$x(t) = \frac{t}{16} - \frac{1}{16 \cdot 4} \operatorname{Sen}(4t) - 20\, U_5(t)\,(t-5) - 100\, U_5(t)$$

$$x(t) = \frac{1}{16}\left[t - \frac{\operatorname{Sen}(4t)}{4}\right] + U_5(t)\left[-20(t-5) - 100\right]$$

TALLER 11

① Encuentre la Solucion de los PVI

ⓐ
$$\begin{cases} y'' + 2y' + 2y = \delta(t-\pi) \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$$

$$\mathcal{L}\{y(t)\} = Y(s)$$

$$\mathcal{L}\{y''\} + 2\mathcal{L}\{y'\} + 2\mathcal{L}\{y\} = \mathcal{L}\{\delta(t-\pi)\}$$

$$s^2 Y(s) - s\,\cancel{y(0)}^{1} - \cancel{y'(0)}^{0} + 2s\,Y(s) - 2\,\cancel{y(0)}^{2} + 2\,Y(s) = e^{-\pi s}$$

$$\cancel{s^2 Y(s)} - s + 2s\,\cancel{Y(s)} - 2 + 2Y(s) = e^{-\pi s}$$

$$Y(s)\left[s^2 + 2s + 2\right] = e^{-\pi s} + s + 2$$

$$Y(s) = \frac{e^{-\pi s}}{(s+1)^2+1} + \frac{s}{(s+1)^2+1} + \frac{2}{(s+1)^2+1}$$

$$\mathcal{L}^{-1}\{Y(s)\} = y(t)$$

$$y(t) = U_\pi(t)\, e^{-t}\, \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} + e^{-t}\, \mathcal{L}^{-1}\left\{\frac{s}{s^2+1} - \frac{1}{s^2+1}\right\}$$

$$+ 2e^{-t}\, \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}$$

$$y(t) = U_\pi(t)\, e^{-t} \cdot \cancel{e^{-\pi}}\, \text{sen}(t-\pi) + e^{-t}\cos(t) - e^{-t}\,\text{sen}\, t)$$

$$+ 2e^{-t}\,\text{sen}(t)$$

$$y(t)\ e^{-t}\left[-U_\pi(t) \cdot \cancel{e^{-\pi}}\, \text{sen}(t) + \cos(t) - \cancel{\text{sen}(t)} + 2\,\text{sen}(t)\right]$$

$$\boxed{y(t) = e^{-t}\left[\text{sen}(t) + \cos(t) - U_\pi(t) \cdot e^{-\pi}\,\text{sen}(t)\right]}$$

ⓑ $y'' + y = \delta(t-\pi)\cos(t)$ $\quad y(0)=0$, $y'(0)=1$

$\mathcal{L}\{y''\} + \mathcal{L}\{y\} = (-1)\,\mathcal{L}\{\delta(t-\pi)\}$

$s^2 Y(s) - s\cancel{y(0)}^{\to 0} - y'(0) + Y(s) = (-1)\,e^{-\pi s}$

$Y(s)\left[s^2+1\right] = 1 - e^{-\pi s}$

$Y(s) = \dfrac{1}{s^2+1} - e^{-\pi s}\dfrac{1}{s^2+1}$ $\qquad \mathcal{L}^{-1}\{Y(s)\} = y(t)$

$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\dfrac{1}{s^2+1}\right\} - \mathcal{L}^{-1}\left\{\dfrac{e^{-\pi s}}{s^2+1}\right\}$

$y(t) = \operatorname{sen}(t) - U_\pi(t)\left[\operatorname{sen}(t-\pi)\right]$

$y(t) = \operatorname{sen}(t) - U_\pi(t)\quad \operatorname{sen} t \cdot \underbrace{\cos(\pi)}_{\to -1} - \cancel{\operatorname{sen}(\pi)}^{\to 0}\cos(t)$

$$\boxed{y(t) = \operatorname{sen}(t) + U_\pi(t)\operatorname{sen} t}$$

ⓒ $y'' + y = U_{\pi/2} + \delta(t-\pi) - U_{3\pi/2}$ $\quad y(0)=0$, $y'(0)=0$

$\mathcal{L}\{y''\} + \mathcal{L}\{y\} = \mathcal{L}\{U_{\pi/2}\} + \mathcal{L}\{\delta(t-\pi)\} - \mathcal{L}\{U_{3\pi/2}\}$

$s^2 Y(s) - s\cancel{y(0)}^{\to 0} - \cancel{y'(0)}^{\to 0} + Y(s) = \dfrac{e^{-\pi s/2}}{s} + e^{-\pi s} - \dfrac{e^{-3\pi s/2}}{s}$

$Y(s)\left[s^2+1\right)$

$$Y(s) = \frac{e^{-\frac{\pi s}{2}}}{s(s^2+1)} + \frac{e^{-\pi s}}{s^2+1} - \frac{e^{-\frac{3\pi s}{2}}}{s(s^2+1)}$$

$$FP = \frac{1}{s} - \frac{s}{s^2+1}$$

$$Y(s) = e^{-\pi s/4}\left(\frac{1}{s} - \frac{s}{s^2+1}\right) + e^{-\pi s}\left(\frac{1}{s^2+1}\right) - e^{-3\pi s/2}\left(\frac{1}{s} - \frac{s}{s^2+1}\right)$$

$$\boxed{\mathcal{L}^{-1}\{Y(s)\} = y(t)}$$

$$y(t) = U_{\pi/4}(t)\left(1 - \cos(t - \pi/4)\right) + U_{\pi}(t)\,\text{sen}(t-\pi) - U_{3\pi/2}\left(1 - \cos(t - 3\pi/2)\right)$$

$$y(t) = U_{\pi/4}(t)\left(1 - \left[\cos t \cdot \cos(\pi/4) + \text{sen}(t)\cdot\text{sen}(\pi/4)\right]\right)$$
$$+ U_{\pi}(t)\left(\text{sen}\,t \cdot \cos\pi - \cancel{\text{sen}(\pi)}^{\to 0}\cdot\cos(t)\right)$$
$$- U_{3\pi/2}\left(1 - \left[\cos(t)\cdot\cancel{\cos(3\pi/2)}^{\to 0} + \text{sen}(t)\cdot\text{sen}(3\pi/2)\right]\right]$$

$$\boxed{y(t) = U_{\pi/4}(t)\left[1 - \left[\frac{\sqrt{2}}{2}\cdot\cos(t) + \text{sen}(t)\frac{\sqrt{2}}{2}\right]\right] - U_{\pi}(t)\,\text{sen}(t) - U_{3\pi/2}\left[1 + [\text{sen}(t)]\right]}$$

d) $ty'' + 2ty' + 2y = 0$; $y(0)=0$
$y'(0)=3$

$\mathcal{L}\{ty''\} + 2\mathcal{L}\{ty'\} + 2\mathcal{L}\{y\} = \mathcal{L}\{0\}$

$-\frac{d}{ds}\left[s^2 \cdot Y(s) - s\cancel{y(0)}^{\to 0} - \cancel{y'(0)}^{\to 3}\right] + 2\frac{d}{ds}\left[sY(s) - \cancel{y(0)}^{\to 0}\right]$

$+2\,Y(s)$

$\frac{-d}{ds}\left[s^2 \cdot Y(s) - 3\right] + 2\frac{d}{ds}\left[s\,Y(s)\right] + 2\,Y(s) = 0$

$-2s \cdot Y(s) - s^2 Y'(s) + 2\cancel{Y(s)} + 2sY'(s) + \cancel{2Y(s)} = 0$

$Y'(s)\left[-s^2 - 2s\right] - 2s \cdot Y(s) = 0$

$\frac{dy}{ds}\left[-s^2 - 2s\right] = 2s\,y$

$\frac{dy}{y} = \frac{-2s}{s(s^2+2)}$

$\int \frac{dy}{y} = \int \frac{-2}{s^2+2}\,ds =$

$\ln y = -2\ln|s^2+2| + c$

$\ln y = \ln(s^2+2)^{-2} + c$

$y = \frac{1}{(s^2+2)^2} \cdot e^c$

$Y(s) = K\,\frac{1}{(s^2+2)^2}$

$$\mathcal{L}^{-1}\{Y(s)\} = y(t)$$

$$y(t) = K\ \mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2}\right\}$$

$$y(t) = K\ e^{-2t}\cdot t$$

$$y'(t) = \left(-2e^{-2t}\,t + e^{-2t}\right)K$$

$$3 = (0 + 1)K$$

$$\boxed{K = 3}$$

$$\boxed{y(t) = 3te^{-2t}}$$

e) $ty'' + y = 0$ $\quad y(0) = 0$, $y'(0) = 0$ $\quad \mathcal{L}\{y(t)\} = Y(s)$

$$\mathcal{L}\{ty''\} + \mathcal{L}\{y\} = \mathcal{L}\{0\}$$

$$-\frac{d}{ds}\left[s^2Y(s) - \cancel{sy(0)}^{0} - \cancel{y'(0)}^{0}\right] + Y(s) = 0$$

$$-\frac{d}{ds}\left[s^2 Y(s)\right]$$

$$-2sY(s) - s^2Y'(s) + Y(s) = 0$$

$$-s^2Y'(s) + Y(s)\left[1-2s\right] = 0$$

$$Y'(s) + Y(s)\left(\frac{2}{s} - \frac{1}{s^2}\right) = 0$$

$$y'(s) + y(s)\,\frac{2s^2 - s}{s^3} = 0$$

$$\frac{dy}{ds} = +y\left[\frac{s - 2s^2}{s^3}\right]$$

$$\int \frac{dy}{y} = \int \frac{s - 2s^2}{s^3}\, ds$$

$$\ln|y| = \int \frac{1}{s^2} - \int \frac{2}{s}$$

$$L\,|y| = -\frac{1}{s} - 2\ln|s|$$

$$y(s) = e^{-\frac{1}{s} - 2\ln(s) + c}$$

$$y(s) = e^{-1/s} \cdot \frac{1}{s^2} \cdot e^{c} \quad K$$

$$y(s) = e^{-1/s} \cdot \frac{1}{s^2}\, K$$

$$\mathcal{L}^{-1}\{y(s)\} = y(t)$$

$$y(t) = \delta\left(t - \frac{1}{s^2}\right) t \cdot K$$

$$y(t) = \delta\left(t - \frac{1}{s^2}\right) t \cdot K$$

$$y'(t) = \frac{1}{s^2}\,\delta'\left(t - \frac{1}{s^2}\right)K - \delta'\left(t - \frac{1}{s^2}\right)K$$

$$y(t) = t \cdot K \;\bullet\; \mathcal{L}\{\cancel{\text{re}}\}$$

(2) Utilice la transformada de una derivada para hallar las siguientes transformadas

(a) $\mathcal{L}\{t^n\} = (-1)^n \frac{d^n}{ds^n} \cdot \mathcal{L}\{1\}$

$= (-1)^n \frac{d^n}{ds^n}\left[\frac{1}{s}\right]$

(b) ~~$\mathcal{L}\{\text{sen}\, t\}$~~

(b) $\mathcal{L}\{\text{sen}^2\, 5t\} = \mathcal{L}\left\{\frac{1-\cos(10t)}{2}\right\}$

$\mathcal{L}\left\{\frac{1}{2}\right\} - \frac{1}{2}\mathcal{L}\{\cos(10t)\}$

$$\boxed{\frac{1}{2s} - \frac{1}{2}\,\frac{s}{s^2+100}}$$

(c) $\mathcal{L}\{t\, e^{5t}\}$ $\quad \left|\frac{d}{ds}\left[\frac{1}{s-5}\right]\right. \quad (s-5)^{-1}$

$-\frac{d}{ds}\mathcal{L}\{e^{5t}\}$ $\quad \left\{-\frac{1}{(s-5)^2}\right\}$

3) Encuentre la solucion del PVI

a) $ty'' - y' = 2t^2 \qquad y(0) = 0$

$$\mathcal{L}\{y(t)\} = y(s)$$

$$-\frac{d}{ds}\left[s^2 y(s) - s\cancel{y(0)} - y'(0)\right] - s\,y(s) + \cancel{y(0)}$$

$$= 2\frac{d^2}{ds^2}\left[\frac{1}{s}\right]$$

$$-\frac{d}{ds}\left[s^2 y(s) - y'(0)\right] - s\,y(s) = 2\,\frac{2}{s^3}$$

$$-2s \cdot y(s) - s^2 y'(s) - s\,y(s) = \frac{4}{s^3}$$

$$-y'(s)\,s^2 + y(s)\left[-2s - s\right] = \frac{4}{s^3}$$

$$-y'(s)s^2 - 3s\,y(s) = \frac{4}{s^3}$$

$$y'(s) + \frac{3}{s}\,y(s) = -\frac{4}{s^5}$$

$$e^{3\int \frac{1}{s}\,ds} = e^{\ln (s)^3} = s^3$$

$$y(s)\cdot s^3 = \int -\frac{4}{s^2} \qquad y(s) = \frac{4}{s^4} + C$$

$$\mathcal{L}^{-1}\{Y(s)\} = y(t)$$

$$y(t) = \frac{4}{6} t^3 + \mathcal{L}^{-1}\{c\}$$

$$y(t) = \frac{4}{6} t^3 + c\,\delta(t)$$

$$0 = c\,\delta(t)$$

$$c = 0$$

$$\boxed{y(t) = \frac{4}{6} t^3}$$

(b) $2y'' + ty' - 2y = 10 \qquad y(0)=0 \quad y'(0)=0$

$$2\mathcal{L}\{y''\} - \frac{d}{ds}\mathcal{L}\{y'\} - 2\mathcal{L}\{y\} = \mathcal{L}\{10\}$$

$$2\left[s^2 Y(s) - s\cancel{y(0)}^{0} - \cancel{y'(0)}^{0}\right] - \frac{d}{ds}\left[sY(s) - \cancel{y(0)}^{0}\right] - 2Y(s) = \frac{10}{s}$$

$$2s^2 Y(s) - \left[Y(s) + sY'(s)\right] - 2Y(s) = \frac{10}{s}$$

$$2s^2 Y(s) - Y(s) - sY'(s) - 2Y(s) = 10/s$$

$$-sY'(s) + Y(s)\left[2s^2 - 1 - 2\right] = 10/s$$

$$Y'(s) + Y(s)\left[\frac{3}{s} - 2s\right] = -\frac{10}{s^2}$$

$$e^{\int \frac{3}{s} - 2s} \to 3\ln(s) - s^2$$

$$s^3 \cdot e^{-2s}$$

$$y(s)\, s^3 e^{-s^2} = \int 10s \cdot e^{-s^2}$$

$\hookrightarrow$ $u = -s^2$

$$y(s)\, s^3 e^{-s^2} = +5 \int e^u$$

$du = -2s$

$$y(s)\, s^3 \cdot e^{-s^2} = +5 e^{-s^2} + C$$

$-\frac{du}{2} = s$

$$y(s) = \frac{+5}{s^3} + \frac{C e^{s^2}}{s^3}$$

$$\mathcal{L}^{-1}\{y(s)\} = y(t)$$

$$y(t) = \frac{+5}{2} t^2 + \mathcal{L}^{-1}\{C e^{s^2} s^{-3}\}$$

1

$$y(t) = +\frac{5}{2} t^2$$

$$y'(t) = +5t$$

$$y''(t) = +5$$

2

$$C\, \delta(t+s)\, \frac{t}{s^4}$$

$$\boxed{y(t) = +\frac{5}{2} t^2 + \frac{Ct}{s^4}\, \delta(t+s)}$$

(C) $ty'' + y = 12t$ $\qquad y(0) = 0$, $y'(0) = 13$

$$\mathcal{L}\{ty''\} + \mathcal{L}\{y\} = \mathcal{L}\{12t\}$$

$$-\frac{d}{ds}\mathcal{L}\{y''\} + \mathcal{L}\{y\} = -\frac{d}{ds}\mathcal{L}\{12\}$$

$$-\frac{d}{ds}\left[s^2 Y(s) - s\cancel{y(0)}^{\to 0} - y'(0)^{\to 13}\right] + Y(s) = -\frac{d}{ds}\left[\frac{12}{s}\right]$$

$$-\frac{d}{ds}\left[s^2 Y(s) - 13\right] + Y(s) = -\frac{12}{s^2}$$

$$-2s\,Y(s) - s^2 Y'(s) + Y(s) = -\frac{12}{s^2}$$

$$2s\,Y(s) + s^2 Y'(s) - Y(s) = \frac{12}{s^2}$$

$$Y'(s)s^2 + Y(s)\left[2s - 1\right] = \frac{12}{s^2}$$

$$Y'(s) + Y(s)\left[\frac{2}{s} - \frac{1}{s^2}\right] = \frac{12}{s^4}$$

$$e^{\int \frac{2}{s} - \frac{1}{s^2}} = e^{\ln s^2 + \frac{1}{s}}$$

$$= s^2 e^{-s}$$

$$Y(s)\, s^2 e^{-s} = \int \frac{12}{s^2}\, e^{-s}$$

$u = s^2$
$du = 2s$

(4) $t^2y'' + t^2y + t^2y = 0$ $t \neq 0$

(5) Encuentre la transformada de laplace de las funciones periodicas siguentes

$$\mathcal{L}\{f(t)\} = \frac{\int_0^T e^{-st} f(t)\,dt}{1-e^{-sT}}$$

ⓐ $f(t+2) = f(t)$

$$f(t) = \begin{cases} 1 & \text{si} \quad 0 \le t \le 1 \\ -1 & \text{si} \quad 1 \le t < 2 \end{cases}$$

Para $0 \le t \le 1$

$f(t) = 1$

$$\frac{\int_0^2 e^{-s}}{1 - e^{-2s}}$$

$$\boxed{\frac{2e^{-s}}{1 - e^{-2s}}}$$

Para $1 \le t < 2$

$f(t) = -1$

$$\frac{\int_0^2 e^{s}}{1 - e^{-2s}}$$

$$\boxed{\frac{2e^{s}}{1 - e^{-2s}}}$$

ⓑ $f(t+\pi) = f(t)$ donde $f(t) = \text{sen}(t)$

$0 \le t < \pi$

transformado

$$\frac{\boxed{\int_0^{\pi} e^{-st} \, \text{sen}(t) \, dt}}{1 - e^{-\pi s}}$$

$$\begin{cases} \text{sen}(t) & 0 \le t \le \pi \\ 0 & t > \pi \end{cases}$$

$\text{sen}(t) \; (\cancel{U_0} - U_{\pi}(t))$ $\to 1$

$\text{sen}(t) - \text{sen}(t) \, U_{\pi}(t)$

$$\mathcal{L}\{\operatorname{sen}(t)\} - \mathcal{L}\{\operatorname{sen}(t)\, u_\pi(t)\}$$

$$\frac{1}{s^2+1} + e^{-\pi s}\frac{1}{s^2+1}$$

$$\boxed{\frac{1}{s^2+1}\left(1+e^{-\pi s}\right)}$$

La transformada de la funcion periodica sera

$$\boxed{\frac{1}{(1-e^{-\pi s})}\cdot\frac{1}{(s^2+1)}\left(1+e^{-\pi s}\right)}$$

(6) Considere los problemas de valor inicial

(a) $y'' + 2y' + 10y = 0$ $\quad y(0)=0$

$y'(0)=1$

$$\mathcal{L}\{y''\} + 2\mathcal{L}\{y'\} + 10\mathcal{L}\{y\} = 0$$

$$s^2 Y(s) - \cancel{s\,y(0)}^{\to 0} - \cancel{y'(0)}^{\to 1} + 2s\,Y(s) - \cancel{2y(0)}^{\to 0} + 10\,Y(s)$$

$$s^2 Y(s) - 1 + 2s\,Y(s) + 10\,Y(s) = 0$$

$$Y(s)\left[s^2 + 2s + 10\right] = 1$$

$$Y(s) = \frac{1}{(s+1)^2 + 9}$$

$$\delta(t)$$

$$\mathcal{L}^{-1}\{Y(s)\} = y(t)$$

$$y(t) = e^{-t}\,\mathcal{L}^{-1}\left\{\frac{1}{s^2+9}\right\}$$

$$\boxed{y(t) = \frac{e^{-t}}{3}\,\operatorname{sen}(3t)}$$

(b) $y'' + 2y' + 10y = \delta(t)$

$y(0) = 0$

$y'(0) = 0$

$$\mathcal{L}\{\delta(t-0)\} = e^{-0s} = 1$$

$$\mathcal{L}\{y''\} + 2\mathcal{L}\{y'\} + 10\mathcal{L}\{y\} = \mathcal{L}\{\delta(t)\}$$

$$\boxed{y(t) = \frac{e^{-t}}{3}\,\operatorname{sen}(3t)}$$

Si son iguales

la (a) fue en el momento antes o después del golpe y (b) fue en el momento exacto del golpe

7) Evalue cada una de las siguientes transformadas de Laplace

(a) $\mathcal{L}\{t^2 * te^t\}$

$$\mathcal{L}\{t^2\} \cdot \mathcal{L}\{te^t\}$$

$$\frac{d^2}{ds^2}\mathcal{L}\{1\} \cdot (-1)\frac{d}{ds}\mathcal{L}\{e^t\}$$

$$\frac{2}{s^3} \cdot (-1)\frac{d}{ds}\left[\frac{1}{(s-1)}\right]$$

$$\frac{2}{s^3} \cdot (-1)(-1)\frac{1}{(s-1)^2}$$

$$\boxed{\frac{2}{s^3(s-1)^2}}$$

(b) $\mathcal{L}\{e^{-t} * e^t \cos t\}$

$$\frac{1}{s+1} \cdot \frac{s}{s^2+1}$$

$$\boxed{\frac{1}{s+1} \cdot \frac{s-1}{(s-1)^2+1}}$$

(c) $\mathcal{L}\{ \int_0^t e^{-\tau} \cos(t-\tau)\, d\tau$

$\mathcal{L}\{ e^{-t} * \cos(t) \}$

$$\boxed{\frac{1}{s+1} \cdot \frac{s}{s^2+1}}$$

(D) $\mathcal{L}\{ \int_0^t \operatorname{sen}\tau \cdot \cos(t-\tau \}$

$\mathcal{L}\{ \operatorname{sen} t * \cos t \}$

$$\frac{1}{s^2+1} \cdot \frac{s}{s^2+1}$$

(8) Use la transformada de laplace para resolver la ecuacion integrodiferencia

(a) $f(t) + \int_0^t (t-\tau) f(\tau)\, d\tau = t$

$\mathcal{L}\{f(t)\} + \mathcal{L}\{ \int_0^t (t-\tau) f(\tau) d\tau \} = \mathcal{L}\{ t \}$

$Y(s) + \mathcal{L}\{ t * f(t) \} = (-)\frac{d}{ds}\mathcal{L}\{1\}$

$$Y(S) + \mathcal{L}\{t\} \cdot \mathcal{L}\{f(t)\} = \frac{1}{S^2}$$

$$Y(S) + \left[+\frac{1}{S^2}\right] \cdot Y(S) = \frac{1}{S^2}$$

$$Y(S)\left[1 + \cancel{\frac{1}{S^2}}\right] = \frac{1}{S^2}$$

$$Y(S) \cdot \frac{S^2+1}{S^2} = \frac{1}{S^2}$$

$$\mathcal{L}^{-1}\{Y(S)\} = \mathcal{L}\left\{\frac{1}{S^2+1}\right\}$$

$$f(t) = \operatorname{senh}(t)$$

(b) $$f(t) = te^t + \int_0^t \tau f(t-\tau)\, d\tau$$

$$\mathcal{L}\{f(t)\} = Y(S)$$

$$Y(S) = \mathcal{L}\{te^t\} + \mathcal{L}\left\{\int_0^t \tau f(t-\tau)\, d\tau\right\}$$

$$Y(S) = (-)\frac{d}{dS}\left(\frac{1}{S-1}\right) + \mathcal{L}\{t * f(t)\}$$

$$Y(S) = \frac{1}{(S-1)^2} + \mathcal{L}\{t\}\,\mathcal{L}\{f(t)\}$$

$$y(s) = \frac{1}{(s-1)^2} + \frac{1}{s^2} y(s)$$

$$y(s) - \frac{y(s)}{s^2} = \frac{1}{(s-1)^2}$$

$$y(s)\left(1 - \frac{1}{s^2}\right) = \frac{1}{(s-1)^2}$$

$$y(s)\left[\frac{s^2-1}{s^2}\right] = \frac{1}{(s-1)^2}$$

$$y(s) = \frac{s^2}{(s^2-1)\,(s-1)^2}$$

$$s^2 = \frac{A}{(s-1)} + \frac{B}{(s-1)} + \frac{Cs+D}{s^2-1}$$

$$s^2 = (s-1)(s^2-1)A + (s-1)(s^2-1)$$

(c)

$$y'(t) = 1 - \text{sen}(t) - \int_0^t y(\tau)\,d\tau \qquad y(0) = 0$$

$$\mathcal{L}\{y'(t)\} = \mathcal{L}\{1\} - \mathcal{L}\{\text{sen}(t)\} - \{\int_0^t y(\tau)\,d\tau\}$$

$$s\,y(s) - \cancel{y(0)}^{\,0} = \frac{1}{s} - \frac{1}{s^2+1} - \frac{y(s)}{s}$$

$$s\,y(s) + \frac{y(s)}{s} = \frac{1}{s} - \frac{1}{s^2+1}$$

$$y(s)\cdot\left[s+\frac{1}{s}\right]=\frac{1}{s}-\frac{1}{s^2+1}$$

$$y(s)\left[\frac{s^2+1}{s}\right]=\frac{1}{s}-\frac{1}{s^2}$$

$$y(s)=\frac{1}{s^2+1}-\frac{1}{s(s^2+1)}$$

$$\hookrightarrow \frac{A}{s}+\frac{Bs+D}{s^2+1}$$

$$y(s)=\frac{1}{s^2+1}-\frac{1}{s}-\frac{s}{s^2+1}$$

$$y(t)=\operatorname{sen}(t)\overset{1}{-}1-\cos(t)$$

(D) $f(t)=\cos(t)+\int_0^t e^{-\tau}f(t-\tau)\,d\tau$

$$\mathcal{L}\{f(t)\}=\mathcal{L}\{\cos(t)\}+\mathcal{L}\left\{\int_0^t e^{-\tau}f(t-\tau)\,d\tau\right\}$$

$$y(s)=\frac{s}{s^2+1}+\mathcal{L}\{e^{-t}*f(t)\}$$

$$y(s)=\frac{s}{s^2+1}+\left[\frac{1}{s+1}\cdot y(s)\right]$$

$$Y(S)\cdot\left(1-\cancel{\frac{1}{S+1}}\right)=\frac{S}{S^2+1}$$

$$Y(S)\cdot\left(\frac{S+1-1}{S+1}\right)=\frac{S}{S^2+1}$$

$$Y(S)=\frac{S(S+1)}{S(S^2+1)}$$

$$Y(S)=\frac{S+1}{S^2+1}$$

$$Y(S)=\frac{S}{S^2+1}+\frac{1}{S^2+1}$$

$$\boxed{Y(t)=\cos(t)+\operatorname{sen}(t)}$$

e) $f(t)=e^t+e^t\int_0^t e^{-t}f(\tau)\,d\tau$

$$\mathcal{L}\{f(t)\}=\mathcal{L}\{e^t\}+\mathcal{L}\{\int_0^t f(\tau)\,d\tau\}$$

$$Y(S)=\frac{1}{S-1}+\frac{Y(S)}{S}$$

$$Y(S)\cdot\left[1-\cancel{\frac{1}{S}}\right]=\frac{1}{S-1}$$

$$Y(S)\cdot\left[\frac{S-1}{S}\right]=\frac{1}{S-1}$$

$$Y(S)=\frac{S}{(S-1)^2}$$

$$Y(S)=\frac{S-1}{(S-1)^2}+\frac{1}{(S-1)^2}$$

$$\boxed{Y(t)=e^t+e^t t}$$

9) Resuelva el modelo para un sistema forzado Masa - resorte con amortiguamiento

$$Mx'' + \beta x' + Kx = f(t)$$

$x(0) = 1$
$x'(0) = 0$

a) $M = \frac{1}{2}$ $\beta = 1$ $K = 5$ y f es la funcion, $f(t+2\pi) = f(t)$
donde

$$f(t) = \begin{cases} 1 & \text{si } 0 \le t \le \pi \\ -1 & \text{si } \pi \le t \le 2\pi \end{cases}$$

$$\frac{1}{2}x'' + x' + 5x = F(t)$$

$$\boxed{\int_0^{2\pi} e^{-st} f(t)\,dt} \over 1 - e^{-\pi s}$$

$$x'' + 2x' + 10x = 2f(t)$$

Transformada de $f(t)$

$$1(U_0 - U_\pi) - 1(U_\pi - U_{2\pi})$$

$$1 - U_\pi(t) - 1(U_\pi)(t) + U_{2\pi}(t)$$

$$F(s) = \frac{1}{s} - e^{-\pi s} - e^{-\pi s} + e^{2\pi s}$$

$$F(s) = \frac{\frac{1}{s} - 2e^{-\pi s} + e^{2\pi s}}{1 - e^{-2\pi s}}$$

(11) Use derivadas de transformadas para encontrar

(a) $\mathcal{L}^{-1}\left\{ \ln\left(\frac{S-3}{S+1}\right) \right\}$ $\to F(s)$

$$\mathcal{L}\{t\,f(t)\} = -\frac{d}{ds} F(s)$$

$$\mathcal{L}\{t\,f(t)\} = -\frac{d}{ds}\left[\ln(S-3) - \ln(S+1)\right]$$

$$\mathcal{L}\{t\,f(t)\} = -\frac{1}{S-3} + \frac{1}{S+1}$$

$$\mathcal{L}^{-1}\mathcal{L}\{t\,f(t)\} = \mathcal{L}^{-1}\left\{\frac{1}{S+1} - \frac{1}{S-3}\right\}$$

$$t\,f(t) = e^{-t} - e^{3t}$$

$$f(t) = \frac{e^{-t}}{t} - \frac{e^{3t}}{t}$$

ⓑ $\mathcal{L}^{-1}\left\{ \tan^{-1} \frac{1}{s} \right\}$

$$\mathcal{L}\{t f(t)\} = -\frac{d}{ds} F(s)$$

$$\mathcal{L}\{t f(t)\} = -\frac{d}{ds}\left[\tan^{-1}\left(\frac{1}{s}\right)\right]$$

$$\mathcal{L}\{t f(t)\} = \frac{1}{\frac{1}{s^2}+\frac{1}{1}} \cdot -\frac{1}{s^2}$$

$$\mathcal{L}\{t f(t)\} = \frac{1}{\frac{1+s^2}{\cancel{s^2}}} \cdot \frac{1}{\cancel{s^2}}$$

$$\mathcal{L}\{t f(t)\} = \frac{1}{s^2+1}$$

$$\mathcal{L}\{t f(t)\} = \frac{1}{s^2+1}$$

$$\mathcal{L}^{-1}\mathcal{L}\{t f(t)\} = \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}$$

$$f(t) = \frac{\operatorname{sen}(t)}{t}$$

ⓒ $\mathcal{L}^{-1}\left\{ \tan^{-1}\left(\frac{3}{s+2}\right) \right\}$

$$\mathcal{L}\{t f(t)\} = -\frac{d}{ds} F(s)$$

$$\mathcal{L}\{t f(t)\} = -\frac{d}{ds}\left[\tan^{-1} \frac{3}{s+2}\right]$$

$$\mathcal{L}\{t f(t)\} = \frac{1}{\left(\frac{3}{s+2}\right)^2 + 1}$$

$$= \frac{1}{\frac{9+(s+2)^2}{\cancel{(s+2)^2}}} \cdot \frac{-3}{\cancel{(s+2)^2}}$$

$$= \frac{3}{9+(s+2)^2}$$

$$= \frac{3}{(s+2)^2+9}$$

$$\mathcal{L}^{-1}\mathcal{L}\{t\, f(t)\} = \mathcal{L}^{-1}\left\{\frac{3}{(s+2)^2+9}\right\}$$

$$t\ f(t) = e^{-2t}\ \mathcal{L}^{-1}\left\{\frac{3}{s^2+9}\right\}$$

$$t\ f(t) = e^{-2t}\ \operatorname{sen}(3t)$$

$$\boxed{f(t) = \frac{e^{-2t}\ \operatorname{sen}(3t)}{t}}$$

(12) Demuestre que

a) $\mathcal{L}\left\{\frac{e^{t}-e^{-t}}{t}\right\} = \ln(s+1) - \ln(s-1)$, $s>1$

$$\mathcal{L}^{-1}\mathcal{L}\left\{\frac{e^{t}-e^{-t}}{t}\right\} = \mathcal{L}^{-1}\left\{\ln\left(\frac{s+1}{s-1}\right)\right\}$$

$$\frac{e^{t}-e^{-t}}{t} = \mathcal{L}^{-1}\left\{\ln\frac{(s+1)}{(s-1)}\right\}$$

$$\mathcal{L}\{t\,(f(t))\} = -\frac{d}{ds}\ \ln\frac{(s+1)}{(s-1)}$$

$$\mathcal{L}\{t\ f(t)\} = -\frac{d}{ds}\ \ln(s+1) - \ln(s-1)$$

$$\mathcal{L}^{-1}\mathcal{L}\{t\ f(t)\} = \mathcal{L}^{-1}\left\{\frac{1}{s-1} - \frac{1}{s+1}\right\}$$

$$f(t) = \frac{e^{t} - e^{-t}}{t}$$

(b) $\mathcal{L}\left\{ \underbrace{\int_0^t \frac{1-\cos(k\tau)}{\tau}\, d\tau}_{f(t)} \right\} = \frac{1}{2s} \ln\left(\frac{s^2+k^2}{s^2}\right)$

$$\frac{1}{2\cancel{s}} \ln\left(\frac{s^2+k^2}{s^2}\right) = \frac{F(s)}{\cancel{s}}$$

$$\mathcal{L}\{t\, f(t)\} = -\frac{d}{ds}\left[\frac{1}{2}\ln(s^2+k^2) - \ln(s^2)\right]$$

$$\mathcal{L}\{t\, f(t)\} = -\frac{1}{2}\left[\frac{2s}{s^2+k^2} - \frac{2s}{s^2}\right]$$

$$\mathcal{L}\{t\, f(t)\} = \boxed{\frac{s}{s^2} - \frac{s}{s^2+k^2}}$$

$$f(t) = \frac{1-\cos(kt)}{t}$$

$$t\, f(t) = \mathcal{L}\{1-\cos(kt)\}$$

$$\boxed{\frac{1}{s} - \frac{s}{s^2+k^2}}$$